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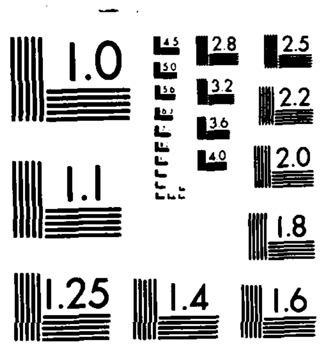

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INTERNAL REFLECTION OF AN ELECTROMAGNETIC WAVE FROM AN ELLIPSOID

BY WALTER P. REID

RESEARCH AND TECHNOLOGY DEPARTMENT

25 MAY 1982

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NAVAL SURFACE WEAPONS CENTER

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A study was made of the process of internal reflection of a polarized electromagnetic wave from an ellipsoid of revolution. The source of the radiation was at one focus and the reflected signal was assumed to strike a target at the second focus. The objective was to select an arrangement that would provide radiation arriving at the test object with electric vectors that are all parallel to each other. This might, perhaps, be achieved by having reflecting plates only on appropriately chosen parts of the ellipsoid. It was found that the only suitable procedure is to permit rays to be reflected just

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from a band around the equator of the ellipsoid, which in turn is essentially equivalent to having an elliptical cylinder as the reflector instead of an ellipsoid.

As a related matter, radiation energy density was investigated. It was found that the intensity of the radiation arriving at the second focus varies greatly with direction even when the source radiates uniformly.

FOREWORD

A study was made of the process of internal reflection of a polarized electromagnetic wave from an ellipsoid of revolution. The source of the radiation was at one focus and the reflected signal was assumed to strike a target at the second focus. The objective was to determine the feasibility of using an arrangement of this type with an airplane at the second focus in order to see how well it could withstand an electromagnetic pulse from lightning or the explosion of an atom bomb. Funding for this work was provided by NAVSEASYSCOM Task Number 24296, Element 62764N.

This report was reviewed by James E. Partak, F32 and Dr. Andrew H. Van Tuyl, R44.

Ira M. Blatstein

IRA M. BLATSTEIN
By direction

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INTERNAL REFLECTION OF AN ELECTROMAGNETIC WAVE FROM AN ELLIPSOID

INTRODUCTION

Electrical systems for power, communication, computing or other purposes can be adversely affected if struck by lightning or some other intense electrical impulse such as that associated with a nuclear explosion. The vulnerability of an airplane, helicopter, ship or other object is sometimes evaluated by subjecting it to electromagnetic radiation under certain test conditions. Strong signals generated by high electromagnetic fields can jump air gaps or cause dielectric breakdown of insulators in components inside these systems and thus travel along paths that are not available to weaker impulses, causing damage or upsetting the system. Therefore a test object must be exposed to radiation of at least moderate intensity in order to determine its ability to withstand lightning or the electromagnetic pulse (EMP) from an atom bomb. In order to minimize the amount of energy that must be generated, and also to avoid interference with facilities in the neighborhood of the test, it is desirable to try to concentrate energy upon the target. It is well known that sound or light generated at one focus of an elliptical enclosure will be reflected to the other focus. For this reason, an analysis is being made here to explore the possibility of using an ellipsoid of revolution as a reflector at an EMP test facility. This treatment is just a cursory feasibility study, and not an exhaustive analysis of all aspects of the problem.

For simplicity, let the ellipsoid be a perfect conductor, and therefore a perfect reflector. Assume throughout that the source is an oscillating dipole at one focus and that the signal generated is polarized in a plane containing the axis of the dipole. The effect of reflection on the polarization and intensity of the radiation will be determined for two different orientations of the dipole axis.

ANALYSIS

Take the equation of the ellipsoid to be

$$\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1 \tag{1}$$

with $a > b$, and with the foci at $F_1 = (f,0,0)$ and $F_2 = (-f,0,0)$, where

$$f^2 = a^2 - b^2. \tag{2}$$

All rays originating at F_1 will arrive at F_2 after a single reflection. Denote the point of reflection as $P = (x_o, y_o, z_o)$ and the distances from foci to P as

ρ_1 and ρ_2.

VERTICAL DIPOLE SOURCE

The first example treated will have the oscillating dipole source perpendicular to the x axis. All such orientations are equivalent because the ellipsoid is a surface of revolution about the x axis. It is sufficient, therefore, to take the particular case in which the dipole is oscillating parallel to the z axis, which in turn will be vertical.

Denote the plane containing the x axis and the point P as

$$z = y \tan \phi. \tag{3}$$

This defines the angle ϕ. The ray from one focus to the other will travel entirely in plane (3).

A unit vector from the source to P is

$$\bar{\nu}_1 = [\bar{i}\,(x_o-f) + \bar{j}\,y_o + \bar{k}\,z_o]\,/\rho_1 \tag{4}$$

where

$$\rho_1 = \sqrt{(x_o-f)^2 + y_o^{\,2} + z_o^{\,2}}. \tag{5}$$

use (1) with this to obtain

$$a\rho_1 = \sqrt{a^2(x_o^{\,2} - 2\,x_o\,f + f^2) + a^2b^2 - b^2x_o^{\,2}} \tag{6}$$

$$= \sqrt{f^2\,x_o^{\,2} - 2\,a^2\,x_o\,f + a^4} \tag{7}$$

$$= a^2 - x_o\,f. \tag{8}$$

A unit vector in the direction of the reflected ray is

$$\bar{\nu}_2 = [\bar{i}\,(x_o + f) + \bar{j}\,y_o + \bar{k}\,z_o]\,/\rho_2 \tag{9}$$

where

$$\rho_2 = \sqrt{(x_o + f)^2 + y_o^{\,2} + z_o^{\,2}}. \tag{10}$$

Replacing f by -f in (8) gives

$$a\rho_2 = a^2 + x_o\,f. \tag{11}$$

Equations (8) and (11) are general relationships that apply in any ellipse. In the work that follows they will occasionally be used to simplify some of the expressions.

Let the vertical plane containing the dipole source and the point P be written as

$$(x-f) \sin \Psi_1 + y \cos \Psi_1 = 0, \tag{12}$$

where Ψ_1 is the angle between this plane and the xz plane. A unit vector normal to plane (12) is

$$\bar{\mu}_1 = \bar{i} \sin \Psi_1 + \bar{j} \cos \Psi_1. \tag{13}$$

The electric vector in the ray going from F_1 to P is assumed to lie in plane (12), and so it must be at right angles to $\bar{\mu}_1$. In addition, since the ray is a transverse wave, the electric vector must also be perpendicular to the direction of propagation, and hence to $\bar{\nu}_1$. Just its amplitude and direction will be considered, but not the variation with time. So, put

$$\bar{E}_1 = (\bar{\mu}_1 \times \bar{\nu}_1) E \tag{14}$$

$$= \{z_o (\bar{i} \cos \Psi_1 - \bar{j} \sin \Psi_1) + \bar{k} [y_o \sin \Psi_1 + (f-x_o) \cos \Psi_1] \} E/\rho_1 \tag{15}$$

$$= [z_o (\bar{i} \cos \Psi_1 - \bar{j} \sin \Psi_1) + \bar{k} y_o \csc \Psi_1] E/\rho_1 \quad \text{for } \Psi_1 \neq 0 \tag{16}$$

$$= [z_o(\bar{i} \cos \Psi_1 - \bar{j} \sin \Psi_1) + \bar{k} (f-x_o) \sec \Psi_1] E/\rho_1 \quad \text{for } \Psi_1 \neq \pi/2. \tag{17}$$

This electric vector lies in the vertical plane (12) and so the radiation is said to be plane polarized. The present objective is to see whether the reflected ray is also polarized in a vertical plane.

As the first step in considering the process of reflection, the component of $\bar{E}$, perpendicular to the ellipsoid at P will be obtained.

Let $\bar{n}$ be a unit vector perpendicular to the ellipsoid. To determine this vector, take the gradient of (1) and divide by the magnitude of that gradient, as follows:

$$\bar{n} = \frac{\bar{i}\, b^2 x_o + a^2 (\bar{j}\, y_o + \bar{k}\, z_o)}{\sqrt{x_o^2 b^4 + a^4 (y_o^2 + z_o^2)}} . \tag{18}$$

But

$$x_o^2\ b^4 + a^4\ (y_o^2 + z_o^2)$$

$$= b^2\ [x_o^2\ b^2 + a^2(a^2 - x_o^2)] \tag{19}$$

$$= b^2\ (a^2 - fX_o)(a^2 + fX_o) \tag{20}$$

$$= a^2\ b^2\ \rho_1\ \rho_2. \tag{21}$$

Hence

$$\bar{n} = \frac{\bar{i}\ b^2\ x_o + a^2(\bar{j}\ k_o + \bar{k}\ Z_o)}{a\ b\ \sqrt{\rho_1\ \rho_2}} . \tag{21}$$

The component $\bar{E}_m$ of the incident electric vector in the direction normal to the ellipsoid can now be found. Thus, from (16) and (22), we have

$$(a\ b\ \rho_1\ \sqrt{\rho_1\ \rho_2})\bar{E}_n$$

$$= (a\ b\ \rho_1\sqrt{\rho_1\ \rho_2})\ (\bar{E}\cdot\bar{n})\ \bar{n} \tag{23}$$

$$= Z_o(b^2\ x_o\ \cos\psi_1 - a^2\ y_o\ \sin\psi_1 + a^2 y_o\ \csc\psi_1)\ E\ \bar{n} \tag{24}$$

$$= Z_o\ \cos\psi_1(b^2\ X_o + a^2\ y_o\ \mathrm{ctn}\ \psi_1)\ E\ \bar{n} \tag{25}$$

$$= Z_o\ \cos\psi_1[b^2\ X_o + a^2(f-X_o)]\ E\ \bar{n} \tag{26}$$

$$= f\ Z_o\ (a^2 - f\ X_o)\ E\ \bar{n}\ \cos\psi_1 \tag{27}$$

$$= a\ \rho_1\ f\ Z_o\ E\ \bar{n}\ \cos\psi_1. \tag{28}$$

Hence

$$\bar{E}_n = \frac{f\ Z_o\ E\ \bar{n}\ \cos\psi_1}{b\sqrt{\rho_1\ \rho_2}} . \tag{29}$$

In the reflection process the component of the electric vector that is parallel to the surface will reverse direction, but the normal component will not. It follows that the parallel components of the incident and reflected rays cancel and the normal components combine. Therefore the reflected field $\bar{E}_2$ is given by

$$\bar{E}_2 = 2\ \bar{E}_n - \bar{E}_1\ . \tag{30}$$

The components of $\bar{E}_2$ will be obtained one at a time by means of this equation along with (16) and (29). Thus.

$$E_{2x} = Z_o \; (2 \; f \; x_o - a \; \rho_2) \; E \cos \psi_1/(a \; \rho_1 \; \rho_2) \tag{31}$$

$$= Z_o \; [2 \; f \; x_o - (a^2 + f \; x_o)] \; E \cos \psi_1/(a \; \rho_1 \; \rho_2) \tag{32}$$

$$= -Z_o \; E \cos \psi_1/ \; \rho_2, \tag{33}$$

$$\begin{aligned} E_{2y} = Z_o \; &(2 \; a \; f \; y_o \cos \psi_1 \\ &+ b^2 \; \rho_2 \sin \psi_1) \; E/(b^2 \; \rho_1 \; \rho_2) \end{aligned} \tag{34}$$

$$\begin{aligned} &= Z_o \; [2 \; a^2 \; f \; (f - x_o) \\ &+ b^2 \; (a^2 + f \; x_o)] \; E \sin \psi_1/(a \; b^2 \; \rho_1 \; \rho_2) \end{aligned} \tag{35}$$

$$\begin{aligned} &= z_o \; [a^2 \; (2 \; f^2 + b^2) \\ &+ f \; x_o \; (b^2 - 2a^2)] \; E \sin \psi_1/(a \; b^2 \; \rho_1 \; \rho_2) \end{aligned} \tag{36}$$

$$= \frac{Z_o \; (a^2 + f^2)(a^2 - f \; x_o) \; E \sin \psi_1}{a \; b^2 \; \rho_1 \; \rho_2} \tag{37}$$

$$= \frac{Z_o \; (a^2 + f^2) \; E \sin \psi_1,}{b^2 \; \rho_2} \tag{38}$$

$$\begin{aligned} E_{2z} = \; &(2 \; f \; a \; z_o{}^2 \sin \psi_1 \cos \psi_1 \\ &- b^2 \; \rho_2 \; y_o) \; E/(b^2 \; \rho_1 \; \rho_2 \sin \psi_1) \end{aligned} \tag{39}$$

$$\begin{aligned} &= [2f \; (a^2 \; b^2 - a^2 \; y_o{}^2 - b^2 \; x_o{}^2) \sin \psi_1 \cos \psi_1 \\ &- b^2 \; y_o(a^2 + f \; x_o)] \; E/(a \; b^2 \; \rho_1 \; \rho_2 \sin \psi_1) \end{aligned} \tag{40}$$

$$= [2f \; (a^2 \; b^2 - a^2 \; y_o{}^2 - b^2 \; x_o{}^2) \cos^2 \psi_1$$

$$+ b^2 (x_o - f)(a^2 + f x_o)]E/(a b^2 \rho_1 \rho_2 \cos \psi_1) \qquad (41)$$

$$= [2 f(a^2 b^2 - a^2 y_o^2 - b^2 x_o^2) \cos^2 \psi_1$$

$$+ b^2(x_o a^2 + x_o^2 f - a^2 f$$

$$-f^2 x_o)(\sin^2 \psi_1 + \cos^2 \psi_1)E/(a b^2 \rho_1 \rho_2 \cos \psi_1) \qquad (42)$$

$$= [b^2(x_o - f)(a^2 + f x_o) \sin^2 \psi_1$$

$$+ (f a^2 b^2 - 2f a^2 y_o^2 - f b^2 x_o^2 + a^2 b^2 x_o$$

$$- b^2 f^2 x_o)\cos^2 \psi_1]E/(a b^2 \rho_1 \rho_2 \cos \psi_1) \qquad (43)$$

$$= [(f a^2 b^2 - f b^2 x_o^2 + a^2 b^2 x_o - b^2 f^2 x_o)\cos^2 \psi_1$$

$$+ (x_o - f)(b^2 a^2 + b^2 f x_o + 2 f^2 a^2$$

$$- 2 f a^2 x_o)\sin^2 \psi_1]E/(a b^2 \rho_1 \rho_2 \cos \psi_1) \qquad (44)$$

$$= [b^2(x_o + f)(a^2 - f x_o) \cos^2 \psi_1$$

$$+ (x_o - f)(a^4 + f^2 a^2 - f^3 x_o^2$$

$$- f a^2 x_o) \sin^2 \psi_1]E/(a b^2 \rho_1 \rho_2 \cos \psi_1) \qquad (45)$$

$$= (a^2 - f x_o) [(x_o - f)(a^2 + f^2) \sin^2 \psi_1$$

$$+ b^2(x_o + f) \cos^2 \psi_1]E/(a b^2 \rho_1 \rho_2 \cos \psi_1) \qquad (46)$$

$$= \frac{[b^2(x_o + f)\cos^2 \psi_1 + (x_o - f)(a^2 + f^2)\sin^2 \psi_1]E}{b^2 \rho_2 \cos \psi_1} \qquad (47)$$

$$= \frac{[b^2(x_o + f) \cos \psi_1 - y_o(a^2 + f^2) \sin \psi_1]E}{b^2 \rho_2} . \qquad (48)$$

Thus the reflected electric vector is

$$\bar{E}_2 = \frac{\{\bar{j} Z_o(a^2 + f^2) \sin \psi_1 - \bar{i} Z_o b^2 \cos \psi_1 + \bar{k}[b^2(x_o + f) \cos \psi_1 - y_o(a^2 + f^2) \sin \psi_1]\} E}{b^2 \rho_2} . \qquad (49)$$

Because the algebraic details involved in obtaining this were lengthy, it is desirable to make independent checks on its correctness. This will be done next.

From (19) and (49) it is easily verified that $\bar{\nu}_2 \cdot \bar{E}_2 = 0$, which shows that $\bar{E}_2$ is perpendicular to $\bar{\nu}_2$, as it should be.

The component of the electric field perpendicular to the ellipsoid should be the same after reflection as it was before. This can be checked from (22) and (9) as follows:

$$\bar{E}_2 \cdot \bar{n} = Z_o E \{a^2[b^2(x_o + f) \cos \psi_1 - y_o(a^2 + f^2) \sin \psi_1]$$

$$+ a^2 y_o(a^2 + f^2) \sin \psi_1 - b^2 x_o \cos \psi_1\} /(a b^3 \rho_2\sqrt{\rho_1 \rho_2}) \qquad (50)$$

$$= Z_o E(a^2 x_o + a^2 f - b^2 x_o) \cos \psi_1/(a b \rho_2\sqrt{\rho_1 \rho_2}) \qquad (51)$$

$$= f z_o E (a^2 + f x_o) \cos \psi_1/(a b \rho_2\sqrt{\rho_1 \rho_2}) \qquad (52)$$

$$= \frac{f Z_o E \cos \psi_1}{b\sqrt{\rho_1 \rho_2}} . \qquad (53)$$

This agrees with (29), and thus provides a second verification of (49).

The magnitude of $\bar{E}_2$ should be E. This can be established by evaluating $\bar{E}_2 \cdot \bar{E}_2$, but the algebraic details in the simplification are tedious. It is much faster to compute the magnitude of E_2 in some numerical cases. The values in Table 1 are offered as a convenience for this purpose or for checking any of the other equations. They have no special significance other than that they are set of integers which are consistent with the work here. That is, they satisfy Eqs. (1), (2), (5), (8), (10), (11), (21) and so on. For example, Eqs. (39) - (48) should all be equal for the particular case given by the numbers in any row of Table 1. (The numerical values to be used for sin ψ_1 and cos ψ_1 follow from (12) and the fact that $0 \leq \psi \leq \pi$). If (49) is not of magnitude E for every example tried, then it is wrong; but the equation has not been shown to be necessarily correct if it does hold. That is, it is necessary for (49) to be satisfied for each set of values in Table 1, but not sufficient. Nevertheless, the fact that (49) is found to be not incorrect for any row of entries in the table will be taken as a

third and final check of the $\bar{E}_2$ in (49).

TABLE 1. SELF-CONSISTE " SETS OF INTEGERS FOR SAMPLE CALCULATIONS

x_o	y_o	Z_o	a	b	f	ρ_1	ρ_2
15	16	0	25	20	15	16	34
15	12	0	25	15	20	13	37
5	6	6	15	9	12	11	19
10	6	3	15	9	12	7	23
15	18	6	35	21	28	23	47
13	14	2	39	15	36	27	51
20	21	12	45	27	36	29	61
30	27	6	55	33	44	31	79

Recall that the source has been taken to be an oscillating dipole with axis parallel to the Z axis so that the electric vector always lies in a vertical plane before reflection. In contrast, it will next be shown that the electric vector in the reflected ray usually does not lie in a vertical plane.

The vertical plane in which the reflected ray travels is the one containing the point of reflection, P, and the second focus F_2.
Write its equation as

$$(x + f) \sin \psi_2 - y \cos \psi_2 = 0. \tag{54}$$

The angle ψ_2 is defined by this equation. A unit vector perpendicular to this plane is

$$\bar{\mu}_2 = \bar{i} \sin \psi_2 - \bar{j} \cos \psi_2. \tag{55}$$

The component of the reflected electric vector perpendicular to plane (54) is

$$\bar{\mu}_2 \cdot \bar{E}_2 = -Z_o[b^2 \cos \psi_1 \sin \psi_2 + (a^2 + f^2) \sin \psi_1 \cos \psi_2]E \tag{56}$$

$$= -Z_o[a^2 (\sin \psi_1 \cos \psi_2 + \cos \psi_1 \sin \psi_2)$$

$$+ f^2(\sin \psi_1 \cos \psi_2 - \cos \psi_1 \sin \psi_2)]E \tag{57}$$

$$= -2 f y_o Z_o(a^2 + f x_o)E/(\sigma_1 \sigma_2) \tag{58}$$

where

$$\sigma_1 = \sqrt{y^2 + (f - x)^2} \tag{59}$$

and σ_2 is the same except that x is replaced by -x. Relation (58) is seen to be zero only when y_o or Z_o is zero, and so only in these special cases will the reflected electric vector be in a vertical plane. It generally has a horizontal component, and can be entirely horizontal - namely when (48) vanishes. We then have

$$E_{2Z} = 0 \text{ when } b^2(x_o + f) \cos \psi_1 = y_o(a^2 + f^2) \sin \psi_1 \tag{60}$$

The angle ψ_1 can be eliminated from this by means of (12), giving

$$E_{2Z} = 0 \text{ when } b^2 x_o^2 + (a^2 + f^2) y_o^2 = b^2 f^2 \tag{61}$$

The locus of all points from which reflection will give a horizontally polarized beam is obtained by solving (1) and (61) simultaneously.

To summarize, then, if the source is a dipole oscillating in the Z direction at a focus of the ellipsoid, then the electric vector in the reflected radiation will almost always have a component perpendicular to the vertical plane in which it is traveling, even though the original signal had no such component. The reflected ray will be polarized in a vertical plane only in the special cases when $\phi = 0$ or $\phi = \pi/2$. Moreover, when (1) and (61) are satisfied, the reflected ray is horizontally polarized.

HORIZONTAL DIPOLE SOURCE

For the second case, let the dipole source at F_1 be oscillating along the x axis. With this orientation the analysis may be more easily handled by first rotating the coordinate axes. However, to facilitate comparison of the present mathematical treatment with the previous one, it seems better to forgo this simplification and be consistent in the method of solution.

Therefore the analysis here will follow the same pattern as before.

Because of the nature of the source, it will be assumed that the electric vector remains in plane (3) as the wave proceeds from the focus to the ellipsoid. A unit vector perpendicular to this plane is

$$\bar{\xi} = \bar{j} \sin \phi - \bar{k} \cos \phi \text{ .} \tag{62}$$

The electric vector in the incident ray is perpendicular to this and also to the vector $\bar{\nu}_1$ in its direction of motion. Thus the amplitude of this vector as it arrives at P from F_1 can be written as

$$\bar{E}_1 = (\bar{\xi} \times \bar{\nu}_1) E \tag{63}$$

$$= [(Z_o \sin \phi + y_o \cos \phi)\bar{i}$$

$$+ (f - x_o)(\bar{j} \cos \phi + \bar{k} \sin \phi)] E / \rho_1 \tag{64}$$

$$= [\bar{i} y_o \sec \phi + (f - x_o)(\bar{j} \cos \phi + \bar{k} \sin \phi)]E/\rho_1 . \tag{65}$$

The component $\bar{E}_n$ of this vector in the direction perpendicular to the ellipsoid is given by

$$(ab\ \rho_1 \sqrt{\rho_1\ \rho_2})\bar{E}_n$$

$$= (a\ b\ \rho_1 \sqrt{\rho_1\ \rho_2})(\bar{E}_1 \cdot \bar{n})\ \bar{n} \tag{66}$$

$$= [b^2\ x_o\ y_o \sec\phi + a^2(f - x_o)(y_o \cos\phi + z_o \sin\phi)]\ E\ \bar{n} \tag{67}$$

$$= y_o \sec\phi[b^2\ x_o + a^2\ (f - x_o)]\ E\ \bar{n} \tag{68}$$

$$= f\ y_o\ (a^2 - f\ x_o)\ E\ \bar{n} \sec\phi, \tag{69}$$

or

$$\bar{E}_n = \frac{f\ y_o\ E\ \bar{n}}{b \sqrt{\rho_1\ \rho_2} \cos\phi}\ . \tag{70}$$

The reflected electric vector $\bar{E}_2$ can now be determined by means of (30). As before, this will be done by getting one component at a time. We then have

$$E_{2x} = y_o\ (2\ f\ x_o - a\ \rho_2)E/(a\ \rho_1\ \rho_2 \cos\phi) \tag{71}$$

$$= y_o\ [2\ f\ x_o - (a^2 + x_o\ f)]E/(a\ \rho_1\ \rho_2 \cos\phi) \tag{72}$$

$$= -\frac{y_o\ E}{\rho_2 \cos\phi}\ , \tag{73}$$

$$E_{2y} = [2\ a\ f\ y_o^2 - b^2\ \rho_2(f-x_o) \cos^2\phi]E/(b^2\ \rho_1\ \rho_2 \cos\phi) \tag{74}$$

$$= [2f(a^2-x_o^2) - (a^2 + f\ x_o)(f-x_o)]E \cos\phi/(a\ \rho_1\ \rho_2) \tag{75}$$

$$= (f\ a^2 - f\ x_o^2 - x_o\ f^2 + x_o\ a^2)E \cos\phi/(a\ \rho_1\ \rho_2) \tag{76}$$

$$= (f + x_o)(a^2 - f\ x_o)E \cos\phi/(a\ \rho_1\ \rho_2) \tag{77}$$

$$= [(f + x_o)\ E \cos\phi]/\rho_2, \tag{78}$$

and $(a\ b^2\ \rho_1\ \rho_2)\ E_{2z}$

$$= [2\ a^2\ f\ y_o\ Z_o - a\ b^2\ \rho_2(f-x_o) \sin\phi \cos\phi]E/\cos\phi \tag{79}$$

$$= [2\ a^2\ f\ y_o^2 \sec^2\phi - b^2(a^2 + f\ x_o)(f-x_o)]E \sin\phi \tag{80}$$

$$= [2 f b^2(a^2-x_o^2) - b^2(a^2 f-a^2 x_o + f^2 x_o - f x_o^2)]E \sin\phi \quad (81)$$

$$= b^2(f a^2 - f x_o^2 + a^2 x_o - x_o f^2)E \sin\phi \quad (82)$$

$$= b^2(f + x_o)(a^2 - f x_o)E \sin\phi. \quad (83)$$

Thus

$$E_{2Z} = [(f + x_o) E \sin\phi]/\rho_2. \quad (84)$$

The reflected electric vector is obtained by combining the results given in (73), (78) and (84), namely:

$$\bar{E}_2 = [-\bar{i} y_o \sec\phi + (f + x_o)(\bar{j} \cos\phi + \bar{k} \sin\phi)]E/\rho_2 \quad (85)$$

This is similar in form to the electric vector before reflection as given in (65). They both lie entirely in plane (3), as can be seen by noting that

$$\bar{\xi} \cdot \bar{E}_1 = \bar{\xi} \cdot \bar{E}_2 = 0 . \quad (86)$$

Thus reflection in this case does not change the plane of polarization.

Equations (65) and (85) are useful forms for the incident and reflected electric vectors for some purposes, but these vectors can be represented more simply if orthogonal unit vectors $\bar{n}$, $\bar{\zeta}$ and an angle λ are used to replace $\bar{i}$, $\bar{j}$, $\bar{k}$ and the angle ϕ. But first the new quantities $\bar{\zeta}$ and λ must be introduced.

From $\bar{\xi}$ of (62) and $\bar{n}$ of (22), define a unit vector $\bar{\zeta}$ as follows:

$$\bar{\zeta} \equiv \bar{\xi} \times \bar{n} \quad (87)$$

$$= [\bar{i} a^2(Z_o \sin\phi + y_o \cos\phi) - \bar{j} b^2 x_o \cos\phi$$

$$-\bar{k} b^2 x_o \sin\phi]/(a b\sqrt{\rho_1 \rho_2}) \quad (88)$$

$$= [\bar{i} a^2 y_o \sec\phi - b^2 x_o(\bar{j} \cos\phi + \bar{k} \sin\phi)]/(ab\sqrt{\rho_1 \rho_2}) \quad (89)$$

This vector is tangent at point P to the curve of intersection of the ellipsoid and plane (3).

Let λ be the angle that the incident ray makes with the normal to the ellipsoid at the point of reflection. Then

$$\cos\lambda = \bar{\nu}_1 \cdot \bar{n} \quad (90)$$

$$= [b^2 x_o(x_o - f) + a^2(y_o^2 + z_o^2)/(ab \rho_1\sqrt{\rho_1 \rho_2}) \quad (91)$$

$$= b^2(a^2-x_o f)/(ab \rho_1 \sqrt{\rho_1 \rho_2}), \quad (92)$$

or

$$\cos\lambda = \frac{b}{\sqrt{\rho_1\ \rho_2}} \tag{93}$$

Thus

$$\sin^2\lambda = 1-\cos^2\lambda = (\rho_1\ \rho_2 - b^2)/(\rho_1\ \rho_2) \tag{94}$$

$$= (a^4 - x_o^{\ 2}\ f^2 - a^2\ b^2)/(a^2\ \rho_1\ \rho_2) \tag{95}$$

$$= f^2\ (a^2 - x_o^{\ 2})/(a^2\ \rho_1\ \rho_2) \tag{96}$$

$$= f^2\ (y_o^{\ 2} + z_o^{\ 2})/(b^2\ \rho_1\ \rho_2) \tag{97}$$

$$= f^2\ y_o^2\ \sec^2\phi/(b^2\ \rho_1\ \rho_2). \tag{98}$$

Hence

$$\sin\lambda = \frac{f\ y_o}{b\cos\phi\ \sqrt{\rho_1\ \rho_2}}\ . \tag{99}$$

From (70) and (99) it is seen that

$$\bar{E}_n = E\ \bar{n}\ \sin\lambda \tag{100}$$

From (65) and (89), we have

$$\bar{E}_1 \cdot \bar{\zeta} = [a^2\ y_o^{\ 2}\ \sec^2\phi - b^2\ x_o(f-x_o)]E/(ab\ \rho_1\sqrt{\rho_1\ \rho_2}) \tag{101}$$

$$= b^2[a^2 - x_o^{\ 2} - x_o(f-x_o)]E/(ab\ \rho_1\ \sqrt{\rho_1\ \rho_2}) \tag{102}$$

$$= bE/\sqrt{\rho_1\ \rho_2}\ =\ E\cos\lambda \tag{103}$$

In similar fashion it is found that

$$\bar{E}_2 \cdot \bar{\zeta} = -E\cos\lambda \tag{104}$$

Therefore

$$\bar{E}_1 = (\bar{n}\sin\lambda + \bar{\zeta}\cos\lambda)E \tag{105}$$

and

$$\bar{E}_2 = (\bar{n}\sin\lambda - \bar{\zeta}\cos\lambda)E \tag{106}$$

These equations show that the electric vectors before and after reflection are in a plane spanned by $\bar{n}$ and $\bar{\zeta}$, as given by (22) and (89). Moreover, this is the plane of polarization of the original wave, namely plane (3), as can be seen by the fact that $\bar{n} \cdot \bar{\xi} = \bar{\zeta} \cdot \bar{\xi} = 0$, where $\bar{\xi}$ is given by (62). Thus reflection here does not change the plane of polarization.

In the case of the vertical dipole source, the incident and reflected electric vectors could also have been expressed in terms of $\bar{n}$ and a tangent vector, as in (105) and (106), (although not with the same $\bar{\zeta}$). However, in that case the

vectors $\bar{n}$, $\bar{\zeta}$ do not span either the plane of polarization or the vertical plane (54) in which the reflected ray travels, and so the $\bar{n}$, $\bar{\zeta}$ do not serve there as a convenient basis for showing the details of reflection.

SIGNAL REINFORCEMENT

It has now been shown that interior reflection from an ellipsoid of revolution usually rotates the plane of polarization of the field generated by a vertical dipole but that there is no rotation of the electric vector when the source is a horizontal dipole. The next problem addressed will be that of determining conditions such that rays arriving at the second focus will have electric vectors that are parallel, so that their effects on the target will reinforce each other. This in turn means finding appropriate locations for reflecting plates. In other words, radiation will not be reflected from the entire ellipsoid, but only from suitably chosen portions of it.

The symbol $\bar{\nu}_2$ denotes a unit vector directed from the point of reflection to the second focus. Use subscripts α and β to designate two different points from which rays are reflected. It will next be shown that the electric vectors in radiation reflected from points α and β can reinforce each other only if each is perpendicular to the plane determined by $\bar{\nu}_{2\alpha}$ and $\bar{\nu}_{2\beta}$.

Assume that $\bar{\nu}_{2\alpha}$ and $\bar{\nu}_{2\beta}$ are not parallel. Then these vectors along with $\bar{\nu}_{2\alpha} \times \bar{\nu}_{2\beta}$ can serve as a three-dimensional basis. Also, it is desired that $\bar{E}_{2\alpha}$ and $\bar{E}_{2\beta}$ be parallel.

Thus

$$\bar{E}_{2\alpha} = \lambda \, \overline{E_{2\beta}} \text{ with } \lambda \neq 0 \tag{107}$$

So, put

$$\bar{E}_{2\alpha} = C_1 \, \bar{\nu}_{2\alpha} + C_2 \, \bar{\nu}_{2\beta} + C_3 \, \bar{\nu}_{2\alpha} \times \bar{\nu}_{2\beta} \tag{108}$$

$$\bar{E}_{2\beta} = \lambda(C_1 \, \bar{\nu}_{2\alpha} + C_2 \, \bar{\nu}_{2\beta} + C_3 \, \bar{\nu}_{2\alpha} \times \bar{\nu}_{2\beta}) \tag{109}$$

But the waves are transverse. That is,

$$\bar{\nu}_{2\alpha} \cdot \bar{E}_{2\alpha} = 0 \quad \text{and} \quad \bar{\nu}_{2\beta} \cdot \bar{E}_{2\beta} = 0 \tag{110}$$

Apply the products of (110) to (108) and (109).

Then, since $\bar{\nu}_2$ is a unit vector, we get

$$0 = C_1 + C_2 \, \bar{\nu}_{2\alpha} \cdot \bar{\nu}_{2\beta}, \tag{111}$$

and

$$0 = \lambda(C_1 \, \bar{\nu}_{2\alpha} \cdot \bar{\nu}_{2\beta} + C_2). \tag{112}$$

But $\lambda \neq 0$. Hence (111) and (112) lead to

$$C_1 = C_2 = 0 \quad \text{unless} \quad \begin{vmatrix} 1 & \bar{\nu}_{2\alpha} \cdot \bar{\nu}_{2\beta} \\ \bar{\nu}_{2\alpha} \cdot \bar{\nu}_{2\beta} & 1 \end{vmatrix} = 0, \tag{113}$$

or

$$C_1 = C_2 = 0 \quad \text{unless} \quad (\bar{\nu}_{2\alpha} \cdot \bar{\nu}_{2\beta})^2 = 1. \tag{114}$$

But $\bar{\nu}_{2\alpha}$ and $\bar{\nu}_{2\beta}$ are not parallel, so that

$$(\bar{\nu}_{2\alpha} \cdot \bar{\nu}_{2\beta})^2 \neq 1 \tag{115}$$

Thus (114), (115) $\Rightarrow$ $C_1 = C_2 = 0,$ (116)

giving

$$\bar{E}_{2\alpha} = C_3 \, \bar{\nu}_{2\alpha} \times \bar{\nu}_{2\beta} \tag{117}$$

and

$$\bar{E}_{2\beta} = \lambda C_3 \, \bar{\nu}_{2\alpha} \times \bar{\nu}_{2\beta} \tag{118}$$

Equations (117) and (118) now show that when (107) applies, the vectors $\bar{E}_{2\alpha}$ and $\bar{E}_{2\beta}$ reflected from any two points α and β are both perpendicular to the plane spanned by $\bar{\nu}_{2\alpha}$ and $\bar{\nu}_{2\beta}$. Since this holds for any two points of reflection, it follows that the electric vectors in rays arriving at the second focus can reinforce each other only in cases where all the incoming radiation is traveling in the same plane, and in addition all the electric vectors must be perpendicular to that plane. This is true in general. The particular cases of two different sources will be considered next.

When the dipole radiator is vertical, it is seen from (49) that (107) will be valid if $Z_o = 0$. That is, all rays reflected from the elliptical band in the xy plane will have electric vectors with only Z components, and will therefore combine constructively. Thus it is sufficient to have $Z_o = 0$. To find necessary conditions, return to (107) and note that all three components of the reflected $\bar{E}_2$ vectors must be parallel. Using just the $\bar{i}$ and $\bar{j}$ components, we have from (49):

$$(-Z_o \cos \psi_1/\rho_2)_\alpha = \lambda \, (-Z_o \cos \psi_1/ \rho_2)_\beta \tag{119}$$

and

$$(Z_o \sin \psi_1/\rho_2)_\alpha = \lambda \, (Z_o \sin \psi_1/\rho_2)_\beta \tag{120}$$

When Z_o is not equal to zero these last two equations may be divided, and so

$$\tan \psi_{1\alpha} = \tan \psi_{1\beta} \qquad \text{if} \quad Z_o \neq 0 \tag{121}$$

or

$$\psi_{1\alpha} = \psi_{1\beta} \qquad \text{if} \quad Z_o \neq 0 \tag{122}$$

Thus, if the incident rays are not in the plane $Z_o = 0$, then they must have the same ψ_1 values, which, from (12), therefore means that they must travel in the same vertical plane. This plane (12) intersects the ellipsoid in a vertical ellipse. But it has already been shown that the reflected rays cannot combine properly unless they lie in the same plane passing through the second focus. Such a plane can intersect the vertical ellipse in at most two points, so that with this possibility no more than two reflected rays will combine with suitable reinforcement. This is inadequate, and so, with a vertical dipole source at one focus the desired combination of reflected, polarized waves at the second focus can be achieved only by restricting reflection to those points in the horizontal band at $Z_o = 0$.

If the dipole source is horizontal, then the reflected electric vector is given by (85). It is seen at once that (107) will hold if $x_o = -f$. That is, for suitable reinforcement it is sufficient to restrict reflection to those parts of the ellipsoid which lie in the plane perpendicular to the x axis at the second focus. This is a circular band with its center at the second focus. To see if there are other possibilities, the three components of the electric vectors in (107) will be equated. At the same time it is convenient to eliminate y_o from the $\bar{i}$ component of $\bar{E}_2$ by means of (54). Thus we have from (85) and (107):

$$[(x_o + f) \sec \phi \tan \psi_2/\rho_2]_\alpha = \lambda[(x_o + f) \sec \phi \tan \psi_2/\rho_2]_\beta \tag{123}$$

$$[(x_o + f) \cos \phi/\rho_2]_\alpha = \lambda[(x_o + f) \cos \phi/\rho_2]_\beta \tag{124}$$

$$[(x_o + f) \sin \phi/\rho_2]_\alpha = \lambda[(x_o + f) \sin \phi/\rho_2]_\beta \tag{125}$$

If x_o is not equal to $-f$, then (125) may be divided by (124).

This gives

$$\tan \phi_\alpha = \tan \phi_\beta \qquad \text{if } x_o \neq -f \tag{126}$$

or

$$\phi_\alpha = \phi_\beta \qquad \text{if } x_o \neq -f \tag{127}$$

using this in (126) or (125) shows that

$$[(x_o + f)/\rho_2]_\alpha = \lambda[(x_o + f)/\rho_2]_\beta \text{ if } x_o \neq -f \tag{128}$$

From (123), (127) and (128), then, it follows that

$$\tan \psi_{2\alpha} = \tan \psi_{2\beta} \qquad \text{if } x_o \neq -f, \tag{129}$$

There are only two rays for which (127) and (129) will hold simultaneously, and one of those rays travels below the plane $Z_o = 0$.

Thus it is seen that if the source is a horizontal dipole at the focus at $x = f$, then signals with electric vectors parallel to each other can arrive at the second focus only if reflection of rays is limited to the portion of the ellipsoid in a circular band at $x_o = -f$. A typical path is shown in Figure 1. All other possibilities are obtained by rotating that pair of rays about the axis through F_1 and F_2.

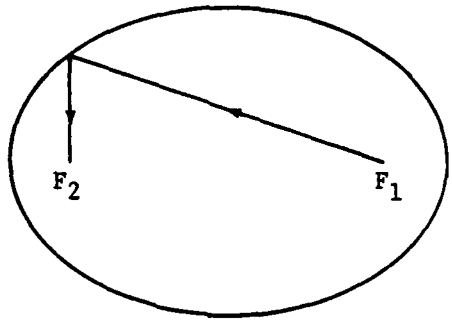

FIGURE 1. THE ONLY TYPE OF PATH THAT IS SUITABLE WHEN THE DIPOLE SOURCE IS HORIZONTAL

INTENSITY CONSIDERATIONS

The energy per unit solid angle I_2 of the radiation arriving at the second focus will be given by

$$I_2 = I_1 \; (\rho_1/\rho_2)^2 \tag{130}$$

where I_1 is the intensity of the source. It is seen that the energy density reaching the target will be less than that generated for reflection from points for which ρ_1 is greater than ρ_2, that is, for reflection from any point on the half of the ellipsoid containing the second focus. In order to show more detailed information, the ρ's will be expressed in terms of the angles involved.

Let α be the angle between the x axis and the direction of a ray, that is, the angle that $\bar{\nabla}$ makes with the x direction. Then, by the law of sines,

$$\rho_1 \sin \alpha_1 = \rho_2 \sin \alpha_2 \tag{131}$$

From the polar form for the equation of an ellipse, it is known that

$$\rho_1 = b^2/(a - f \cos \alpha_1) \tag{132}$$

and

$$\rho_2 = b^2/(a - f \cos \alpha_2) \tag{133}$$

Elimination of the ρ's from these equations leads to

$$(1 - e \cos \alpha_1)/\sin \alpha_1 = (1 - e \cos \alpha_2)/\sin \alpha_2 \tag{134}$$

where $e = f/a$ is the eccentricity. If this equation is squared, it can be solved for $\cos \alpha_2$ or $\sin \alpha_2$. However, in order to made it easier to get these quantities, another relationship will be derived next.

The distance between the foci is 2f and the sum of the lengths of ρ_1 and ρ_2 is 2a. Hence

$$\rho_1 \cos \alpha_1 + \rho_2 \cos \alpha_2 = 2f = 2ea \tag{135}$$

$$= e(\rho_1 + \rho_2) \tag{136}$$

or

$$\rho_1(\cos \alpha_1 - e) = \rho_2(e - \cos \alpha_2) \tag{137}$$

From (131) and (137), then,

$$(\cos \alpha_1 - e)/\sin \alpha_1 = (e - \cos \alpha_2)/\sin \alpha_2 \tag{138}$$

Division of (134) by (138) now eliminates the sines of α_1 and α_2, and from the result it is found that

$$\cos \alpha_2 = \frac{2e - (e^2 + 1) \cos \alpha_1}{1 + e^2 - 2e \cos \alpha_1} \tag{139}$$

This is more convenient than either (134) or (138) for obtaining α_2 from α_1. In addition, by interchanging the subscripts, it can be used for calculating α_1 corresponding to a given α_2.

Eliminition of ρ_2/ρ_1 from (130) by means of (131) gives

$$I_2/I_1 = (\sin \alpha_1/\sin \alpha_2)^2 \ . \tag{140}$$

This serves to determine the intensity ration when the two angles α_1 and α_2 are known. However, an alternate equation can also be obtained by first eliminating $\cos \alpha_1$ from (134) and (138) to get

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{1 - e^2}{1 + e^2 - 2e \cos \alpha_2} \tag{141}$$

This leads to

$$\frac{I_2}{I_1} = \left(\frac{1 - e^2}{1 + e^2 - 2e \cos \alpha_2} \right)^2 \tag{142}$$

The advantage of this form is that it does not contain α_1. For any eccentricity, the intensity ratio can now be calculated as a function of the angle at which the radiation arrives at the second focus. In order to see the general behavior without calculation, it is convenient to first rewrite the equation as follows:

$$\frac{I_2}{I_1} = \left(\frac{b^2}{a^2 + f^2 - 2\ a\ f \cos \alpha_2} \right)^2 \tag{143}$$

Now let

$$C_2{}^2 = a^2 + f^2 - 2\ a\ f \cos \alpha_2 \tag{144}$$

Then

$$I_2/I_1 = (b/C_2)^4 \tag{145}$$

where C_2 is the distance $0Q_2$ of Figure 2 in which 0 is the center of the ellipse, F_1 P F_2 is the path of the radiation, and F_2 Q_2 is of length a. From (130) and (145) an alternate expression for C_2 is seen to be

$$C_2{}^2 = b^2\ \rho_1/\rho_2 \tag{146}$$

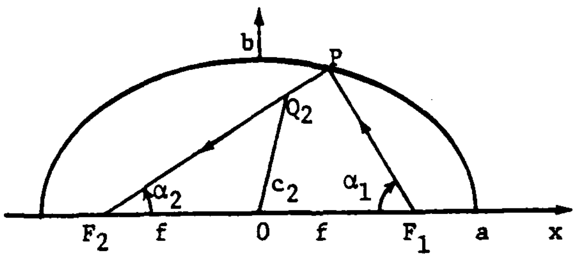

FIGURE 2. DIAGRAM USED TO SHOW THE DISTANCE C_2 THAT ENTERS IN INTENSITY CALCULATIONS

In order to show graphically the behavior indicated by (145), the portion of Figure 2 containing F_2, 0 and Q_2 has been redrawn in Figure 3. The locations of all positions that Q_2 can have are shown by the circle of radius a with center at F_2. Since b in (145) is a constant, it follows that I_2 varies inversely as the fourth power of the distance C_2 in Figure 3. When the angle F_2 0 Q_2 is 90°, the length of C_2 is b, and I_2 and I_1 are equal. The value of C_2 steadily increases from a - f when $\alpha_2 = 0°$ to a + f when $\alpha_2 = 180°$. Thus, the intensity ratio decreases monotonically as α_2 increases from 0° to 180°. The maximum value is $[b/(a-f)]^4 = (a^2-f^2)^2/(a-f)^4$, or

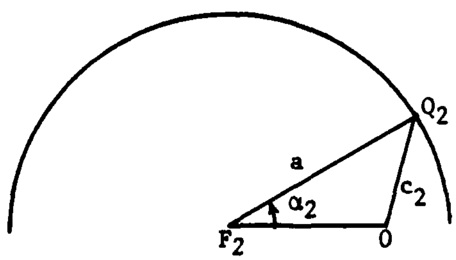

FIGURE 3. DIAGRAM ILLUSTRATING THE RANGE OF VALUES FOR C_2

$$\text{Maximum } I_2/I_1 = [(a + f)/(a - f)]^2 \tag{147}$$

$$= [(1 + e)/(1 - e)]^2 \text{ (when } \alpha_2 = 0) \tag{148}$$

In a similar fashion, it is found that

$$\text{Minimum } I_2/I_1 = [(1 - e)/(1 + e)]^2 \text{ (when } \alpha_2 = 180^\circ) \tag{149}$$

The subscripts 1 and 2 in (143) can be interchanged to give

$$I_2/I_1 = [(a^2 + f^2 - 2\ a\ f \cos \alpha_1)/b^2]^2 \tag{150}$$

By defining C_1 as in (144), this can be written as

$$I_2/I_1 = (C_1/b)^4, \tag{151}$$

with C_1 as the distance $0\ Q_1$ of Figure 4 in which $F_1\ Q_1$ is of length a. Also, comparison of (145) and (151) shows that

$$b^2 = C_1\ C_2, \tag{152}$$

and

$$I_2/I_1 = (C_1/C_2)^2 \tag{153}$$

This form is not as useful as (145) or (151) because both C_1 and C_2 vary.

As an illustration, consider the ellipses shown in Figures 5 and 6. They have eccentricities $e = \sqrt{2}/2$ and $e = \sqrt{3}/2$ respectively, which lead to the following ratios when used in (148) and (149):

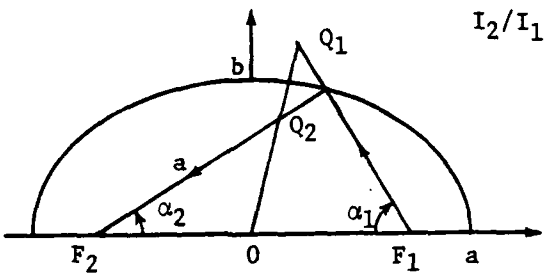

FIGURE 4. DIAGRAM USED TO SHOWN THE DISTANCE C_1 THAT ENTERS IN INTENSITY CALCULATIONS.

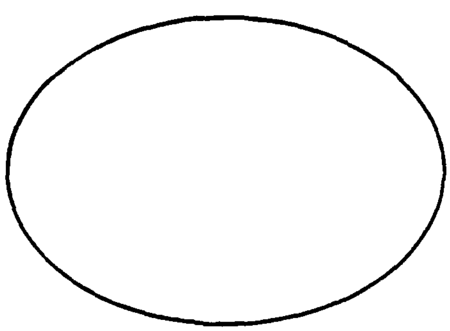

FIGURE 5. A 45° ELLIPSE OF ECCENTRICITY $\sqrt{2}/2$.

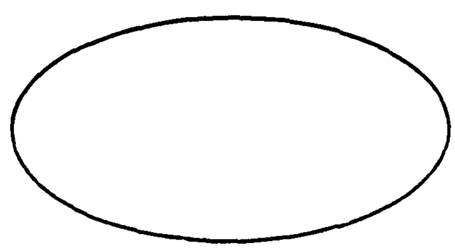

FIGURE 6. A 30° ELLIPSE OF ECCENTRICITY $\sqrt{3}/2$.

With $e = \sqrt{2}/2$ (Fig. 5),

$$\text{Max. } I_2/I_1 = 17 + 12\sqrt{2} \approx 33.97 \tag{154}$$

$$\text{Min. } I_2/I_1 = 17 - 12\sqrt{2} \approx 0.029 \tag{155}$$

With $e = \sqrt{3}/2$ (Fig. 6),

$$\text{Max. } I_2/I_1 = 97 + 56\sqrt{3} \approx 194.0 \tag{156}$$

$$\text{Min. } I_2/I_1 = 97 - 56\sqrt{3} \approx 0.00515 \tag{157}$$

These extremes of intensity ratios occur when α_1 is 180° and 0°, respectively, namely in the axial direction. Thus they have some relevance to a vertical dipole at the first focus because there would in general be a maximum of emitted radiation in these two directions. However, a horizontal dipole source at F_1 will send no radiation along the x axis, and so I_2 will be zero at $\alpha_2 = 0$ rather than a maximum there. In order to determine more about what happens in this case, it is necessary to postulate a radiation pattern for the emitter. Let that choice be

$$I_1 = I_m \sin^2 \alpha_1 . \tag{158}$$

Then (141), (142) and (158) lead to

$$I_2 = I_m \left(\frac{1 - e^2}{1 + e^2 - 2e \cos \alpha_2} \right)^4 \sin^2 \alpha_2 \tag{159}$$

This would be the intensity of the signals arriving at F_2 at an angle α_2 with the x axis if the source at F_1 were a horizontal dipole radiating in accordance with (158).

The intensity prescribed in (158) for the source has a maximum value of I_m at $\alpha_1 = 90°$. However, for the radiation arriving at F_2, it is found that

$$\left.\begin{array}{c} I_2 \text{ of (159) is a maximum} \\ \text{when } \cos \alpha_2 = (\sqrt{1 + 34 e^2 + e^4} - 1 - e)/4e \end{array}\right\} \tag{160}$$

For the shapes illustrated in Figures 5 and 6, Eq. (160) gives the following:

$$\left.\begin{array}{c} \text{With } e = \sqrt{2}/2,\ I_2 \text{ of (159) has} \\ \text{a maximum value of } 14.6\ I_m \text{ when } \alpha_2 = 11.5° \end{array}\right\} \tag{161}$$

$$\left.\begin{array}{c} \text{With } e = \sqrt{3}/2,\ I_2 \text{ of (159) has a} \\ \text{maximum value of } 82.1\ I_m \text{ when } \alpha_2 = 4.8° \end{array}\right\} \tag{162}$$

Thus, with a reflector obtained by revolving the ellipse of Figure 5 or Figure 6 about its major axis, and with a source at one focus radiating with intensity distributed as in (158), there will be a strong concentration of energy arriving at the second focus at a small angle with the central axis, as indicated in (161) and (162).

It was shown above that when the source is a horizontal dipole, only rays following paths of the type shown in Figure 1 will arrive at F_2 with parallel electric vectors. That is, the desired reinforcement of incoming radiation occurs only with those rays coming in to the second focus at an angle $\alpha_2 = 90^o$. From (159) it follows that the intensity of the radiation reaching the target will then be

$$I_2 = I_m \left(\frac{1 - e^2}{1 + e^2} \right)^4 \qquad \text{with } \alpha_2 = 90^o \tag{163}$$

This assumes that the energy is emitted in accordance with (158). With the ellipsoidal shapes of Figures 5 and 6, for example, the values calculated by means of (163) are:

$$\text{With } e = \sqrt{2}/2,\ I_2 = I_m/81 \text{ when } \alpha_2 = 90^o \tag{164}$$

$$\text{With } e = \sqrt{3}/2,\ I_2 = I_m/2401 \text{ when } \alpha_2 = 90^o \tag{165}$$

Thus the radiation intensity I_2 with this arrangement is very small and so the focussing method of Figure 1 is unsatisfactory.

SUMMARY AND CONCLUSIONS

The purpose of this analysis was to see if it would be desirable to use a prolate ellipsoid of revolution to reflect electromagnetic radiation generated at one focus over to a target at the second focus. The major axis of the ellipsoid was taken to be horizontal, and the source was assumed to be an oscillating dipole with its axis either vertical and therefore perpendicular to the major axis, or else along the axis and thus horizontal. Any other orientation would be equivalent to a combination of these two.

The electric vector generated by the vertical dipole was assumed to lie in a vertical plane. It was found that the electric vectors in the radiation arriving at the second focus would on the other hand, have all possible directions, each depending on the point of reflection. The incident and reflected electric vectors in this case are given by (17) and (49).

When the axis of the dipole source is horizontal, the electric vector upon reflection remains in the plane containing the incident and reflected rays. Here (65) and (85) represent the vectors before and after reflection. Both vectors lie in plane (3). Alternate forms for the same two vectors are provided by (105) and (106). From these last equations it is seen that the radiation pattern of the electric vectors arriving at the second focus is the same as that generated.

It was shown that electric vectors coming in to F_2 can reinforce each other only if the rays are traveling in the same plane and the electric vectors are perpendicular to that plane. With a vertical dipole source, this means that reflection of rays should be restricted to the horizontal, elliptical band in the x y plane. This reflecting surface should be part of a cylinder rather than an ellipsoid because the dipole source is not a point, but has finite length in the z direction. On the other hand, when the source is a horizontal dipole along the x axis, it was shown that suitable combination of more than two electric vectors can occur only from rays that are reflected from points lying in the plane $x = -f$. A representative ray of this type is shown in Figure 1.

The angular distribution of the energy striking the target is very lopsided. The energy approaches F_2 primarily in a cone that usually makes a small angle with the x axis. When the source is a vertical dipole this maximum incoming energy is along the axis. In this case the ratio of the intensities arriving at the second focus in opposite directions along this major axis would be $(194)^2$ to 1 for the shape shown in Fig. 6 even though the source sends out radiation uniformly in all horizontal directions.

Energy considerations show that the focussing arrangement of Figure 1 is unsatisfactory because the energy density of the signal arriving as shown there is very small in general. For example, with the shape shown in Figure 6 and with a horizontal dipole emitting radiation according to (158), the intensity of the rays directed as in Figure 1 is only $I_m/2401$. By way of comparison, the maximum intensity of the electromagnetic radiation coming in to F_2 in that situation is seen by (162) to be 82 I_m.

Thus, if it is essential that the radiation impinging upon the object at the second focus all be uniformly polarized in the same direction, then the source should be a vertical oscillator, and reflection of the rays must be limited to a band around the ellipsoid in the x y plane. In addition, it would be better to have the focussing done by a portion of an elliptical cylinder rather than by part of an ellipsoid.

The use of an ellipsoid of revolution as a reflecting surface is recommended for any application that could make good use of the very intense radiation that would strike the target on the side facing the source. However, this should be an experiment in which it was not necessary to have a polarized wave impinging on the test object.

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